Screw dynamo and the generation of nonaxisymmetric magnetic fields

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(Received 18 February 1997)

A mechanism is presented here for the amplification of large-scale nonaxisymmetric magnetic fields as a manifestation of the dynamo effect. We generalize a result on restrictions of dynamo actions due to laminar flow originally derived by Zeldovich, Ruzmaikin, and Sokolov [*Magnetic Fields in Astrophysics* (Gordon and Breach, New York, 1983)]. We show how a screwlike motion having ϕ and z components of velocity can help to grow a magnetic field. This model postulates a large-scale flow having ϕ and z components with radial dependences (helical flow). Shear in the radial field, because of a near-flux-freezing condition, causes amplification of the ϕ component of the magnetic field. The radial and axial components grow due to the presence of turbulent diffusion. The shear in the large-scale flow induces an indefinite growth of magnetic field without the α effect; nevertheless, turbulent diffusion forms an important part in the overall mechanism.
[S1063-651X(97)04808-3]

PACS number(s): 47.65.+a, 91.25.Cw

I. INTRODUCTION

The maintenance of large-scale magnetic fields has long been a problem, particularly in astrophysical situations. In literature this is known as ''the Dynamo problem.'' The effect has been ascribed traditionally to the action of a turbulent dynamo; large-scale differential rotation in the disk acting on the poloidal field leads to the generation of the toroidal field and the α effect due to small-scale cyclonic turbulence completes the cycle, converting the toroidal flux to a poloidal flux (Parker [1] and Zeldovich, Ruzmaikin, and Sokolov [2]). As an alternative, Spencer and Cram [3] raised the intriguing possibility that large-scale flows like galactic winds can act as efficient dynamos. On the basis of a local calculation (where only z derivatives of various quantities are kept), they speculated that magnetic fields can grow on a time scale associated with the shear in the wind. However, it is well known that any simple velocity field with the stream lines confined to two-dimensional surfaces can at best act as a slow dynamo, with the growth rate going to zero as the magnetic Reynolds number increases to infinity. In this work we therefore reexamine the dynamo generation of magnetic fields due to large-scale flows like galactic wind or accretion to form a galaxy, without making the local approximation.

In Sec. II, we first begin by showing the general result that any dynamo action based on laminar flows along two-dimensional surfaces will be a slow dynamo. We then mathematically formulate a model dynamo problem where radial fluid motion is neglected compared to the vertical and rotational motions, and the velocities are assumed to be independent of the direction perpendicular to the azimuthal plane. This model has all the essential features of the realistic situation obtained in a typical astrophysical situation, e.g., the wind dynamo, at the same time being analytically tractable. In our proposed mechanism, growth depends on axial and azimuthal flows that are steady and axisymmetric. It should also be noted that an axisymmetric magnetic field cannot be sustained by an axisymmetric velocity field. However, there is no theoretical restriction on the sustainance of a nonaxisymmetric magnetic field by an axisymmetric flow (Cowling [4]). Therefore, we look for nonaxisymmetric solutions to the induction equation in the magnetohydrodynamics kinetic regime which have harmonic dependences for ϕ and z components. The resulting one-dimensional problem is then amenable to a WKBJ solution. Indeed, the problem we are solving is the screw dynamo problem already studied in the literature (see, e.g., Ponomerenko [5]; see also Gilbert [6], Ruzmaikin, Sokoloff, and Shukorov [7], and Soward [8]). However our analytical treatment differs in some essential details from existing works, like that of Ref. [7].

II. DYNAMO GENERATION OF FIELDS DUE TO LARGE SCALE MOTION

Since we are interested in producing a dynamo mechanism which can operate even when the α effect is weak or absent, we work with the classical induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{2.1}$$

where η is the effective diffusion coefficient (including the effect of turbulent diffusion). We cannot afford to neglect turbulent diffusion as it is much stronger than ordinary diffusion, and our model will crucially depend upon it. So, mathematically, the problem reduces to one in which one has to solve this equation with proper boundary conditions with a prescribed velocity field. Before doing this let us examine the general restrictions on the nature of the dynamo models, with laminar velocity fields. Some simple examples of such restrictions are already described in Ref. [2]. We generalize these restrictions further.

A. Restrictions on dynamo models with laminar flows

Let us consider a two-dimensional flow, in which the fluid is moving over a system of statianary surfaces $\psi(\mathbf{r}) = \text{const}$. Following Ref. [2], we decompose the total

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magnetic field into components perpendicular and parallel to the surface $\psi=$const. We have $\mathbf{B}_\perp=(\mathbf{B}\cdot\hat{\mathbf{n}})\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}=\nabla\psi/|\nabla\psi|$ is the normal to the surface. $\mathbf{B}_\parallel$ can be further split up into its solenoidal and irrotational parts:

$$\mathbf{B}_\parallel=\nabla\times(\Phi'\hat{\mathbf{n}})+\nabla\lambda \tag{2.2}$$

Hence

$$\begin{aligned}\mathbf{B}&=\nabla\times(\Phi'\hat{\mathbf{n}})+(\mathbf{B}\cdot\hat{\mathbf{n}})\hat{\mathbf{n}}+\nabla\lambda\\&=\nabla\times\Phi'\frac{\nabla\psi}{|\nabla\psi|}+\mathbf{B}\cdot\nabla\psi\frac{\nabla\psi}{|\nabla\psi|^2}+\nabla\lambda=\nabla\times[\Phi\nabla\psi]\\&\quad+\mathbf{B}_\psi\frac{\nabla\psi}{|\nabla\psi|}+\nabla\lambda,\end{aligned} \tag{2.3}$$

where $\Phi=\Phi'/|\nabla\psi|$ and $B_\psi=\mathbf{B}\cdot\nabla\psi$. Since $\nabla\cdot\mathbf{B}=0$, we have $\nabla\cdot[\mathbf{B}_\psi(\nabla\psi/|\vec{\nabla}\psi|)]=-\nabla^2\lambda$. We substitute this into the induction equation. We write down the induction in the following way:

$$\frac{\partial B_i}{\partial t}=\epsilon_{ijk}\partial_j(\mathbf{V}\times\mathbf{B})_k+\eta\nabla^2\boldsymbol{B}_i. \tag{2.4}$$

We take the dot product of the above equation with $\nabla\psi$ to obtain the following equation:

$$\begin{aligned}\frac{d}{dt}\int\tfrac{1}{2}B_\psi^2d^3\mathbf{r}=&-\eta\int[(\nabla B_\psi)^2+2B_\psi\partial_kB_i(\partial_i\partial_k\psi)\\&+B_\psi B_k\partial_i(\partial_i\partial_k\psi)d^3\mathbf{r}]-\int B_\psi^2\frac{d\ln\rho}{dt}d^3\mathbf{r}.\end{aligned} \tag{2.5}$$

We use equation of continuity to write the last term. Thus, B_ψ can be advected or compressed by the velocity field, but can only be coupled to the other components through diffusion. In other words, generation of B_ψ can only be due to the diffusion of the other components of the field. The growth of this component will then be slow, on the diffusion time scale, and will be proportional to the inverse of the magnetic Reynolds number.

Let us now consider the diffusion dependent terms in more detail. It is quite clear that, as discussed in Ref. [2], growth will depend on the nature of the surfaces $\psi=$const. We also recognize that if the fluid is confined to move over a planer surface, then the magnetic field will inevitably decay as the integral will be negative definite. We turn now to the equation for Φ. Substituting the expression of $\mathbf{B}$ into the induction equation and taking the cross product with $\hat{n}$, we obtain an equation for Φ, remembering that the term $\mathbf{V}\cdot\nabla\psi=0$, since there is no component of velocity perpendicular to the surface:

$$\nabla\psi\times\nabla\frac{d\Phi}{dt}=-\frac{\partial B_\psi}{\partial t}-\nabla\frac{\partial\lambda}{\partial t}+S, \tag{2.6}$$

where S contains terms with λ and terms proportional to η. From this equation, by inverting the operator $\nabla\psi\times\nabla$ (which can be done in Fourier space) we can construct a time evolution equation for Φ:

$$\left(\frac{\partial}{\partial t}+\mathbf{V}\cdot\nabla\right)\Phi=L^{-1}\left[-\frac{\partial B_\psi}{\partial t}-\nabla\frac{\partial\lambda}{\partial t}+S\right] \tag{2.7}$$

or

$$\frac{d\Phi}{dt}=L^{-1}\left[-\frac{\partial B_\psi}{\partial t}-\nabla\frac{\partial\lambda}{\partial t}+S\right], \tag{2.8}$$

where L^{-1} is the operator inverse of $\nabla\psi\times\nabla$. Thus we see that in the time evolution equation of Φ, there is no shear term, and the source is a function of B_ψ, which again grows on the slow-time scale of diffusion; thus, Φ can grow only on the time scale of diffusion. This is an extension of the basic results as given in Ref. [2] (see Chap. 7, pp. 84–86). Thus any flow which is confined to a surface can at best act as a slow dynamo with the growth of the magnetic field occurring on the diffusive time scale. This restriction was not taken into account by Spencer and Cram, since they made a local approximation and thereby obtained a fast dynamo (see Ref. [3]) Sec. 2, Eq. 15; one should obtain $\gamma\to0$ on putting each $\tau\to\infty$, i.e., equivalently setting diffusion coefficients to zero, but γ does not reduce to 0). In our opinion, a similar problem also exists in the work of Chiba and Lesch [9], who considered the dynamo generation of magnetic fields in a disk galaxy with motions confined to the plane of the disk.

Having established that a laminar flow along surfaces can act at best as a slow dynamo, it will suffice to take a simple form for the surface $\psi=$const, along which the fluid flows. Such a simplification allows us to estimate the growth rates analytically, and also to understand qualitatively the magnetic-field generation in a more general situation. We will therefore consider below the case of an axisymmetric flow along a cylinder $\psi=r=$const, where we use a cylindrical coordinate system (r,ϕ,z), with the plane $z=0$ coinciding with the midplane.

B. Mathematical formulation of the problem

We consider the situation where the flow is along cylinders $r=$const, and is axisymmetric. We then take $V_r=0$, $V_\phi=r\omega(r)$, and $V_z=V(r)$. We also assume that there is no z dependence on V_z and ω.

Since the given velocity field is ϕ and z independent, each ϕ and z mode will evolve separately, hence we look for solutions of the form $\mathbf{B}=\mathbf{b}(r)\exp(im\phi+ikz+\Gamma t)$; i.e., each Fourier mode in ϕ and z will evolve independently. Thus m can be regarded as a measure of nonaxisymmetry. At this point, one should recall the fact that an axisymmetric velocity field cannot sustain an axisymmetric magnetic field (Cowling's theorem) [4]. Thus we could not obtain any growth for the $m=0$ mode in view of Cowling's theorem. We see below that this holds explicitly for the growth rates that we derive.

The above set of equations reduces to

$$\frac{\eta}{r}\frac{d}{dr}\left(r\frac{db_r}{dr}\right)-\frac{\eta m^2 b_r}{r^2}-\eta k^2 b_r-i\omega m b_r-ikvb_r-\Gamma b_r-\frac{\eta b_r}{r^2}=\frac{2im\eta b_\phi}{r^2}, \quad (2.9)$$

$$\frac{\eta}{r}\frac{d}{dr}\left(r\frac{db_\phi}{dr}\right)-\frac{\eta m^2 b_\phi}{r^2}-\eta k^2 b_\phi-\frac{\eta b_\phi}{r^2}-\Gamma b_\phi-i\omega m b_\phi-ikvb_\phi=-\frac{2i\eta m b_r}{r^2}-rb_r\frac{d\omega}{dr}. \quad (2.10)$$

We do not use the z component of the induction equation directly; instead we use the divergence-free condition to find out B_z:

$$\nabla\cdot\mathbf{B}=\frac{1}{r}\frac{\partial(rB_r)}{\partial r}+\frac{1}{r}\frac{\partial B_\phi}{\partial\phi}+\frac{\partial B_z}{\partial z}=0. \quad (2.11)$$

We see explicitly that our proposed dynamo model will be a slow dynamo-growth rate $\Gamma\to 0$ as $\eta\to 0$, since diffusion is the only source for r and z components.

We also notice that for $m=0$, i.e., for an axisymmetric magnetic field, the B_r equation is decoupled from B_ϕ; there is no source term, and so no growth, and hence our equations automatically satisfy Cowling's theorem [4]. To solve these equations we resort to the WKBJ approximation methods. We follow Mestel and Subramanian [10] (hereafter MS) in solving these equations.

For the sake of convenience, we scale the above equations to give them dimensionless forms in the following way: we scale $\Gamma\to(r_o^2/\eta)\Gamma$ and $v\to(r_o/\eta)v$, where r_o is any suitable length scale. Hence Eqs. (2.9) and (2.10) reduce to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{db_r}{dr}\right)-\frac{m^2 b_r}{r^2}-k^2 b_r-i\omega m b_r-ikvb_r-\Gamma b_r-\frac{b_r}{r^2}=\frac{2imb_\phi}{r^2}, \quad (2.12)$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{db_\phi}{dr}\right)-\frac{m^2 b_\phi}{r^2}-k^2 b_\phi-\frac{b_\phi}{r^2}-\Gamma b_\phi-i\omega m b_\phi-ikvb_\phi=-\frac{2imb_r}{r^2}-rb_r\frac{d\omega}{dr}. \quad (2.13)$$

C. WKBJ solutions for growth rates

It is convenient to use a new radial coordinate x defined by $r=e^x$ (as in the WKBJ method, one looks for an exponentially decaying solution with the independent variable $\to\pm\infty$ (see, e.g., Heading [11] or MS). With this, Eqs. (2.12) and (2.13) reduce to

$$\frac{d^2b_r}{dx^2}-(m^2+1)b_r-b_re^{2x}(k^2+i\omega m+ikv+\Gamma)=2imb_\phi, \quad (2.14)$$

$$\frac{d^2b_\phi}{dx^2}-(m^2+1)b_\phi-b_\phi e^{2x}(k^2+i\omega m+ikv+\Gamma)=-2imb_r-b_re^{2x}\frac{d\omega}{dx}. \quad (2.15)$$

We rewrite the above equations as

$$\frac{d^2Q}{dx^2}+aQ+bP=0, \quad (2.16)$$

$$\frac{d^2P}{dx^2}+cQ+dP=0, \quad (2.17)$$

where

$$Q=b_r, \quad (2.18)$$

$$P=b_\phi, \quad (2.19)$$

$$a=-(m^2+1)-e^{2x}(k^2+i\omega m+ikv+\Gamma), \quad (2.20)$$

$$b=-2im, \quad (2.21)$$

$$c=2im+e^{2x}\frac{d\omega}{dx}, \quad (2.22)$$

$$d=-(m^2+1)-e^{2x}(k^2+i\omega m+ikv+\Gamma). \quad (2.23)$$

The coefficients a, b, c, and d are assumed to vary with x over a typical scale $L\gg 1$ (i.e., the original physical variables vary over a scale larger than the length scale r_o). Solutions are sought of the forms

$$P=\exp[i\psi(x)][A_o(x)+A_1(x)/L+\cdots], \quad (2.24)$$

$$Q=\exp[i\psi(x)][B_o(x)+B_1(x)/L+\cdots], \quad (2.25)$$

where L is the length scale over which coeffecients a, b, c, and d are assumed to vary, $\psi'\sim\emptyset(L^o)$, $\psi''\sim\emptyset(L^{-1})$, etc., $A_o'\sim\emptyset(L^{-1})$, $A_o''\sim\emptyset(L^{-2})$, etc.

Following MS,

$$(\psi_\pm)^2=p_\pm=[a+d\pm\sqrt{(a+d)^2-4(ad-bc)}]/2 \quad (2.26)$$

or

$$(\psi'_\pm)^2=p_\pm=-(m^2+1)-e^{2x}(k^2+i\omega m+ikv+\Gamma)\pm\left[-2im\left(2im+e^{2x}\left|\frac{d\omega}{dx}\right|\right)\right]^{1/2}, \quad (2.27)$$

where $\psi'_\pm=\pm[(\psi')^2]^{1/2}$. We want to draw the attention of the reader to the fact that Eq. (2.27) is equivalent to (but not quite the same as) Eq. (10) of Ref. [7]. We present a short discussion about this below [see following Eq. (2.32) of this paper].

The general solution of Eqs. (2.24) and (2.25) (as long as the WKBJ treatment is valid) can be written as

$$P=A^{+}\left[K_1\exp\left(i\int_{x_o}^{x}p_{+}^{1/2}dx\right)+K_2\exp\left(-i\int_{x_o}^{x}p_{+}^{1/2}dx\right)\right]$$
$$+A^{-}\left[K_3\exp\left(i\int_{x_o}^{x}p_{-}^{1/2}dx\right)+K_4\exp\left(-i\int_{x_o}^{x}p_{-}^{1/2}dx\right)\right], \tag{2.28}$$

$$Q=B^{+}\left[K_1\exp\left(i\int_{x_o}^{x}p_{+}^{1/2}dx\right)+K_2\exp\left(-i\int_{x_o}^{x}p_{+}^{1/2}dx\right)\right]$$
$$+B^{-}\left[K_3\exp\left(i\int_{x_o}^{x}p_{-}^{1/2}dx\right)+K_4\exp\left(-i\int_{x_o}^{x}p_{-}^{1/2}dx\right)\right]. \tag{2.29}$$

The general ways of analyzing WKBJ solutions whose singular points are in the complex plane was nicely described by Heading [11]. Recall that our solution has to die out at $x\rightarrow\pm\infty$.

Now since there will be complex zeros from $p_{\pm}=0$, we have to choose those nearest to the x axis, so that the solution obeys boundary conditions. Since $p_{\pm}$ goes to zero not in the real axis, but in the complex plane, p_{+} and p_{-} will generally not be zero at the same points. Now our general solution may consist of (i) only p_{+}, (ii) only p_{-}, or (iii) a linear combination of both. Now, since p_{+} and p_{-} do not go to zero simultaneously, and our solution has to die out at $x\rightarrow\pm\infty$, if we have a linear combination of p_{+} and p_{-}, both of them will not die out at $x\rightarrow\pm\infty$, and hence the third option is ruled out. This forces us to set either K_3 and K_4 or K_1 and K_2 to be zero, i.e., a general solution cannot be a combination of p_{+} and p_{-} solutions. We immediately realize that P and (also Q) obeys a one-dimensional Schrödinger-like equation of the form

$$\frac{d^2P}{dx^2}+p_{\pm}(x)P=0. \tag{2.30}$$

The WKBJ solution of this is

$$P=\frac{A}{p_{\pm}^{1/4}}\exp\left(i\int^{x}p_{\pm}^{1/2}dx\right)+\frac{B}{p_{\pm}^{1/4}}\exp\left(-i\int^{x}p_{\pm}^{1/2}dx\right), \tag{2.31}$$

where A and B are constants.

The eigencondition is given by

$$\int_{x_1}^{x_2}p_{\pm}^{1/2}dx=(2n+1)\pi/2, \tag{2.32}$$

where n is an integer, and x_1 and x_2 are the zeros of $p_{\pm}$. We pause here for a while and compare our results with that of Ref. [7]. We draw the attention of the readers to the fact that Ruzmaikin, Sokoloff, and Shukorov made their asymptotic expansions in terms of the conventional radial coordinate r, and solved the radial equation directly using the WKBJ approximation, whereas we worked with equations written in terms of the modified radial variable x. The discrepancies in the final results appear because of that. We, however, feel that the former approach is not quite appropriate. In this context we refer to the book by Heading [11] (pp. 127–131), and references therein, and also to MS and Jeffreys [12] (pp. 245–247). Even Gutzwiller [13] briefly refers to the problem in direct application of the WKBJ method to the radial equation in the hydrogen atom problem in quantum mechanics (see p. 212 of Ref. [13]) and as a remedy refers to Langer [14]. In fact, the transformation $r=\exp(x)$ was not only a convenient one but was also a necessity. We also see that if we work out everything with $\omega=\omega_o/(\sqrt{1+r^2})$ (this type of profile qualitatively resembles galactic rotation curve), then, the prescription of Ref. [7], one finds that there is no extrema of the function $(1/r)(d\omega/dr)$ at any finite r. Thus the method of Ref. [7] does not seem to be the right one for this kind of situation.

We notice that the only growth term is the shear term in the ϕ equation; the r component grows due to the diffusion of the ϕ component. Since growth will be controlled by shear (the $r|d\omega/dr|$ term), the growing modes will naturally be concentrated around the maximum of shear. Thus in that region, we can neglect the diffusion term (the m^2i/r^2 term) compared to the shear term.

Thus we obtain

$$\frac{p_{\pm}}{r^2}=-[k^2+\Gamma]-\frac{m^2+1}{r^2}\pm\frac{1-i}{r}\sqrt{2m}\left[r\left|\frac{d\omega}{dr}\right|\right]^{1/2}$$
$$-i(\omega m+kv).$$

We expand $[r(d\omega/dr)]^{1/2}$ about the point r_m where it is maximum, and retain up to the first nonvanishing order. Thus we obtain (in terms of the r coordinate)

$$\frac{p_{\pm}}{r^2}=A(r-r_m)^2+B(r-r_m)+C, \tag{2.33}$$

where

$$A=-\frac{i}{2}\frac{d^2}{dr_m^2}(\omega m+kv)\pm\frac{1-i}{r_m}\sqrt{2m}\frac{d^2}{d{r_m}^2}\left[r_m\left|\frac{d\omega}{dr_m}\right|\right]^{1/2},$$

$$B=-i\frac{d}{dr_m}(m\omega+kv),$$

$$C=-[k^2+\Gamma]-\frac{m^2+1}{{r_m}^2}-i(m\omega+kv)_{r_m}$$
$$\pm\frac{1-i}{r_m}\sqrt{2m}\left[r_m\left|\frac{d\omega}{dr_m}\right|\right]^{1/2}.$$

Now we recognize that this being a quadratic in complex r, it will have two roots (in general complex), unlike the case in the complex x coordinate [the reason is quite obvious: $r=\exp(x)=\exp(x+2\pi ni)$]. So we need not worry about choosing the correct zeros. We also see that as $r\rightarrow\infty$, $\mathrm{Im}(\int_{z_o}^{z}\sqrt{p_{\pm}})\rightarrow 0$; thus the anti-Stokes line approaches the x axis as $r\rightarrow 0$, obeying the boundary condition.

Thus the eigenvalue Γ is given by

$$\frac{c-\frac{B^2}{4A}}{\sqrt{A}}=n+\tfrac{1}{2}. \tag{2.34}$$

We notice that $B^2<0$. It is obvious that Γ_{p_+} is always greater than Γ_{p_-}. We also notice that when $(m\omega+kv)=0$ identically [case (i); see below], the growth rate for p_- is negative, which signifies decay. So we work with p_+ henceforth. We can have two situations: (i) when $m\omega+kv=0$; when ω and v_z show exactly the same dependence on r, i.e., when the pitch of a helical streamline does not depend on the radius, certain m and k always exist such that

$$\frac{m}{k}=-\frac{v(r)}{\omega(r)}=\text{const},$$

and hence the advection term vanishes identically. (ii) When $m\omega+kv\neq 0$—this the general case.

In the second case, the growth rate reduces to

$$\Gamma=-\eta k^2-\eta\frac{m^2+1}{r_m^2}+\eta\frac{1-i}{r_m}\sqrt{2m}\left[r_m\left|\frac{d\omega}{dr_m}\right|\right]^{1/2}-i(m\omega+kv)|_{r_m}$$
$$+\frac{\left[\frac{d}{dr_m}(m\omega+kv)\right]^2}{2\left(-i\frac{d^2}{dr_m{}^2}(m\omega+kv)+2\frac{1-i}{r_m}\sqrt{\eta}\sqrt{m}\frac{d^2}{dr_m^2}\left[r_m\left|\frac{d\omega}{dr_m}\right|^{1/2}\right]\right)}-(n+\tfrac{1}{2})(x^2+y^2)^{1/4}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right), \tag{2.35}$$

where

$$x=\frac{\sqrt{\eta}}{2}\sqrt{\frac{m}{r_m}\frac{d^2}{dr_m{}^2}\left[r_m\left|\frac{d\omega}{dr_m}\right|\right]^{1/2}},$$

$$y=-\frac{\sqrt{\eta}}{2}\frac{d^2}{dr_m{}^2}(m\omega+kv)-\frac{\sqrt{\eta}}{2}\sqrt{\frac{2m}{r_m}\frac{d^2}{dr_m{}^2}\left[r_m\left|\frac{d\omega}{dr_m}\right|\right]^{1/2}},$$

and

$$\theta=\tan^{-1}\frac{y}{x}.$$

We see from this expression that Γ is negative (i.e. decays) when $m=0$, directly manifesting Cowling's theorem, i.e., an axisymmetric magnetic field cannot be sustained by an axisymmetric velocity field. Also, Γ decreases as $-m^2$ and grows as $\sqrt{m}$; so even though low-m modes will be growing, asymptotically Γ will decrease with increasing m and n—which is also expected from energy consideration; as with increasing m and n, the mode becomes more kinky, i.e., higher currents are associated with the loops, and so needs higher energy to grow. So there will be few growing modes with low-m values, and all higher modes will decay. Please note that our model is essentially a simplified model, a model concerned with the underlying basic principles, representing the real life situations somewhat qualitatively. As a next step one may try to work out the same for a conical flow geometry, even though our basic results will be valid for that case. We also see from the expression of Γ that, in the limit $\eta\to 0$, $\Gamma\to 0$, which shows that our dynamo model is a slow dynamo as expected on theoretical grounds.

III. SUMMARY

In this work we have shown that any velocity field confined to move over a stationary surface will lead to a slow dynamo only; that growth will be on the time scale of diffusion. The component normal to the stationary surfaces has diffusion of the other components only as it source. We mentioned the fact that an axisymmetric magnetic field cannot be sustained by an axisymmetric velocity field—which is known as ''Cowling's theorem'' in the literature. Keeping this constraint in mind, we demonstrated how an axisymmetric cylindrical velocity flow can act as a slow dynamo to produce a nonaxisymmetric magnetic field. We explicitly demonstrated that the growth rate $\Gamma\to 0$ as diffusion coefficient η and azimuthal mode number $m\to 0$, thus directly proving that it is a slow dynamo, and directly satisfying Cowling's theorem. We have also given a physical reason that generation will be confined to the region where shear is dominant. In these context, we compare our procedure with that of Ref. [7].

Our simple model of dynamo generation of a magnetic field may be applied in some realistic astrophysical systems. In the case of galactic wind, taking over from the disk in the form of a helical motion, our model may be applied to explain the growth of magnetic field. At this stage one should probably try to justify the existence of turbulent diffusion in the absence of on α effect (molecular diffusion will be too weak to cause any significant growth of the magnetic field). Recently, it has been theorized that the α effect is suppressed long before large-scale magnetic-field strength could reach the present μG level (Kulsrud and Anderson [15]). Now the relevant question for our case is whether turbulent diffusion is also suppressed along with the α effect. In this context we refer to the results of recent works of Gruzinov and Diamond [16] (see also Vainshtein *et al.* [17], and Jones and Galloway [18]), where it was shown that the α effect may be supressed, but not turbulent diffusion. This supports our assumptions.

ACKNOWLEDGMENTS

A major part of the work has been done while the author was visiting NCRA, TIFR, Pune, India. For that the author wishes to thank Vijay Kapahi, Director, NCRA, and Kandaswamy Subramanian, NCRA, for enabling him to visit NCRA. The author is also thankful to Kandaswamy Subramanian for drawing his attention to this novel problem and for many fruitful discussions. The author wishes to thank Apratim Chatterji and Indranil Mitra for a careful reading of the manuscript.

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